Efficiency Model Based On Response Surface Methodology for A 3 Phase Induction Motor Using Python

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Abstract

This study conjures the development of a mathematical model to scrutinize the influence of slip and voltage to frequency ratio on the efficiency of the Induction Motor. Response Surface Methodology based efficiency models are developed using the Full Factorial Design (FFD) of experiments. Initially an FFD based layout plan to carry out the experimentation is prepared. Response surface based efficiency model is prepared and checked against the results of the experiments carried out. To form the model, various statistical calculations have to be performed. These calculations are carried out using number crunchers like Python. The statistical module in the scipy library is used for this purpose. To further validate the model, additional experiments with random control factors are carried out, and the accuracy of the model is confirmed. The importance of voltage to frequency ratio is underlined in this study.

Keywords: Response Surface Methodology, Induction Motor Efficiency, voltage to frequency ratio, python, scipy statistics.

I. Introduction

In the present day industry, the major systems that consume electrical energy are the drive systems. More importantly, the induction motor (IM) is the most widely used electric motor which is used for a vast range of applications like pumps, compressors, elevators, lathes, conveyors etc [1]. Hence, the scenario demands that the operation of induction motors should be as efficient as possible, as it will lead to significant energy savings [4].

Since the efficienct operation of induction motors pays rich dividends in terms of energy savings, in a time when every unit of energy conserved is important, it becomes imperative to better understand the effects that various factors will have on its efficiency. So it is necessary to have an accurate relationship between the efficiency of the IM and the control factors so as to analyze the efficiency curves. This in turn, calls for a set of experiments on the IM drive. We need to run the IM at different configurations of it's control factors, and measure its efficiency for each configuration. By doing this, the behavior of the machine is better understood, and appropriate decisions can be made so as to have maximum efficiency operation at all times [10].

The experiments can be conducted by varying one control factor at a time and keeping the other factors unchanged. This approach is usually very time consuming, especially if the control factors are on the higher side. Also, the number of trials to be run increases with the increase in the number of control factors. Another disadvantage of this approach is that the interactive effects among the control factors is not reflected in the results of the experiments carried out by this approach. So, a better approach would be to develop an appropriate mathematical model which shows the relation between the efficiency and the various control factors [3].

RSM is the preferred modeling technique of researchers as it is fairly simple to build an RSM model. It requires minimum knowledge of the process and requires fewer trials. This makes it a less time consuming and cost effective means of experimentation. An added advantage is that the interaction effects between the control factors can be obtained from RSM based models.

Systematic planning of the experiments needs to be done before developing the RSM model. Design Of Experiments (DOE) approach is used for this purpose. This enables for a well planned experimentation process, reducing time and cost. The efficient set of experimental sampling points is defined by using DOE, and the response of the system is observed at only these points. Depending on the levels of input variables, the combinations at which the response is observed is formulated by DOE [6].

The objective of this study is to develop RSM based mathematical models that accurately predict the efficiency of the IM with voltage to frequncy ratio and slip as control factors by conducting experiments as per full factorial design (FFD) of DOE.

II. Design of Experiments (DOE) and Response Surface Methodology (RSM)

When we talk about any experiment, the basic elements we should know about are the factors (control variables), levels (settings) and the response (outcome) of the experiment. The factors or control variables are those variables which of the system or process that govern the response of the system. The response of the system is usually a quality characteristic of a process, for example, cost, torque, power etc.

A. Design of Experiments (DoE)

Design of Experiments or DoE is a powerful tool that can be used in a variety of experimental situations. DoE allows for multiple input factors to be manipulated determining their effect on a desired output (response). By manipulating multiple inputs at the same time, DoE can identify important interactions that may be missed when experimenting with one factor at a time. The experimental design systematically defines the efficient set of experimental sampling points at which the responses must be computed or observed. The DoE takes levels of input variables to formulate the different combinations at which the outputs are observed or computed. In DOE, each factor is defined with distinct number of levels (2 levels, 3 levels, 4 levels..)

Based on whether we test all possible combinations or a part of the combinations, DoE can broadly be classified as

- Full Factorial Design or
- Fractional Factorial Design.

The experiments carried out for this study were done using the Full Factorial Design (FFD) where the response is measured for every combination of the control factor.

B. DoE design

For this investigation, five levels for the voltage to frequency ratio (V/f) and percentage slip (% s) were selected as depicted in Table I. Care was taken through initial experimentation to ensure that the IM operates within its limits. Subsequently, the sample points for the experiments were deterimened as per Full Factorial Design (FFD) of the Design Of Experiments (DOE) approach.

TABLE I: Factors And Their Levels

Control Factor	Unit	Control Levels				
		Lev1	Lev2	Lev3	Lev4	Lev5
V/f	-	7	7.3	7.6	8	8.4
s	%	0.67	1.6	3.33	4.33	5.0

C. Response Surface Methdology (RSM)

Once we are done with the selection of specific sampling points for a particular experiment, the next step is to develop a model based on RSM. Response surface methodology (RSM) is a collection of mathematical and statistical techniques for empirical model building. By careful design of experiments, the objective is to optimize a response (output variable) which is influenced by several independent variables (input variables). The application of RSM to design optimization is aimed at reducing the cost of expensive analysis methods.

D. Induction Motor Specifications

The experimental load tests were carried out on a 3-phase, 415 V, 3.7 KW, 1430 RPM squirrel-cage IM. For each configuration of the control factors V/f and slip, the input and output powers of the IM were noted, and the efficiency was calculated. Table II presents the experimental layout plan as per FFD. This describes the sample points at which experimental trials will be carried out.

E. Development of mathematical model and estiamtion of regression coefficients

A non-linear second order mathematical model has been developed to show the effect of voltage to frequency ratio and slip on the IM efficiency. The response surface equation for two control factors considered in the present case is

$$\eta = b_0 + b_1(V/f) + b_2 s + b_{11}(V/f)^2 + b_{22}s^2 + b_{12}(V/f)s \quad (1)$$

where, $b_1, b_2, ...b_{12}$ are regression coefficients of linear, quadratic and interaction terms of the model [11] were determined by

$$b = (X^T X)^{-1} X^T Y \quad (2)$$

In Eqn. (2), b is the matrix of parameter estimates, X is the calculation matrix, Y is the matrix of computed values of efficiency and T represents transpose operator. The mathematical model as determined by the regression analysis to predict

TABLE II: Experimental layout plan as per FFD

Trial No.	Control Factor Settings	
	V/f	$Slip(\%)$
1	7.0	0.67
2	7.0	1.66
3	7.0	3.33
4	7.0	4.33
5	7.0	5.00
6	7.3	0.67
7	7.3	1.33
8	7.3	3.33
9	7.3	4.33
10	7.3	5.00
11	7.6	0.67
12	7.6	1.33
13	7.6	3.33
14	7.6	4.33
15	7.6	5.00
16	8.0	0.67
17	8.0	1.66
18	8.0	3.33
19	8.0	4.33
20	8.0	5.00
21	8.4	0.67
22	8.4	1.66
23	8.4	3.33
24	8.4	4.33
25	8.4	5.00

the efficiency of IM is given by

$$\eta = -801.67 + 4.37(V/f) + 31.86s - 0.005(V/f)^2 - 1.43s^2 - 0.05(V/f)s \tag{3}$$

III. Python used for model development

Equations (1), (2) and 3 depict the model which is to be designed so that the efficiency of any control level can be predicted. The libraries made use of are:

- Numpy: for handling arrays
- Scipy: for statistical calculations
- Pylab: for plotting various graphs and comparisons

Based on the model developed, predicted efficiencies for each combination of control factors were obtained. These predicted efficiency values are compared with the actual efficiency values obtained experimentally, and their absolute errors are found and tabulated as shown in Table III.

The adequacy of model has been verified by the value of co-efficient of regression (R^2) which is used to test the goodness of fit of the developed model. It is the proportion of the variation in the dependent factor explained by the model. The closer the value of co-efficient of regression (R^2) to unity, better is the fit of the model. In the present study, the R^2 value was found to be 0.78 which indicates a good correlation between the observed and predicted values of IM efficiency.

IV. Determining Model Acuracy

The efficiency predictive model (Eq. 3) was used to test the accuracy of the developed model using both the experimental data results and verification tests. Table III depicts the comparison of predicted and experimental efficiency values as per FFD. It can be observed from Table III that the efficiency values predicted by the RSM model closely agree with that of experimental values. The maximum prediction error was found to be 8.15%.

To validate the model further, another set of experiments were carried out for the different random values of V/f and slip. This layout is depicted in Table IV. The experimantally calculated values are also shown. Fig. 1 shows the shows the plot of percentage errors between the calculated efficiency and predicted efficiency for the set of validation experiments. As can be observed, the maximum percentage error is about 8% which is well within the allowable limits.

TABLE III: Experimental Efficiency Vs. Predicted Efficiency and Absolute Errors

Trial No.	Efficiency (%)		Error
	Experimental	Predicted	% Error
1	61.3006	65.6716	-7.1305
2	77.3440	74.6144	3.5291
3	83.1448	82.6527	0.5919
4	80.9521	82.1989	-1.5401
5	82.7844	79.8045	3.5996
6	70.9796	2.2475	-1.7862
7	83.9216	80.3413	4.2663
8	86.0596	86.8004	-0.8608
9	83.6505	85.4337	-2.1317
10	83.6122	82.4277	1.4167
11	79.8521	76.5331	4.1565
12	81.5627	83.7779	-2.7159
13	85.2286	88.6578	-4.0236
14	84.8005	86.3783	-1.8606
15	81.7684	82.7607	-1.2135
16	82.8096	78.6846	4.9813
17	87.8909	84.7975	3.5196
18	84.0402	87.5718	-4.2022
19	83.3653	84.0751	-0.8514
20	81.7369	79.6421	2.5629
21	70.9796	76.7645	-8.1501
22	82.0315	81.7455	0.3486
23	85.8493	82.4142	4.0013
24	80.2179	77.7004	3.1383
25	70.2651	72.4519	-3.1122

TABLE IV: Trials for validation of model

Trial No.	Control Factor Settings		Calculated Efficiency
	V/f	$Slip(\%)$	$\eta(\%)$
1	7.1	4.00	80.4250
2	7.1	3.00	83.4898
3	7.3	3.33	89.7964
4	7.3	3.66	89.1482
5	7.4	3.33	85.0143
6	7.4	1.33	79.3162
7	7.8	2.00	74.4036
8	7.8	2.33	86.9498
9	8.2	1.30	70.5033
10	8.2	2.60	89.0246

V. Results and Discussion

The efficiency characteristic of IM at any point in the range of the designed experiment can be forecasted by substituting the values of voltage to frequency ratio and slip in Eqn. (3). Fig. 2 shows the response surface plot which illustrates the interaction effect of voltage to frequency ratio and slip on % efficiency of IM.

It is clear from Fig. 2 that the efficiency of IM demonstrates nonlinear behavior with both voltage to frequency ratio and slip. In addition, it is seen that minimum efficiency occurs at low values of slip and voltage to frequency ratio. Also, the IM efficiency is highly sensitive to change in slip at lower range of values of voltage to frequency ratio.

The direct effect plot showing the variations of IM efficiency with variations slip for three different hold values of voltage to frequency ratio is depicted in Fig. 3. It is seen from Fig. 3 that the efficiency is at it's optimum value for all values of slip when IM is operating with a voltage to frequency ratio of 7.6. Further, it is observed that maximum efficiency occurs at a lower slip as the voltage to frequency ratio is increased. The useful operating region showing operating efficiency is illustrated in the contour plot of Fig. 4.

From the above discussions, it can be accepted that RSM modeling approach is very useful in analyzing the influence of multiple parametres on the operation of IM. Using the concepts of DOE, we can minimize the number of trial experiments to be conducted. The drawback of this approach observed during this study is that RSM modeling is more accurate when the range of control parameters selected is narrow. The interpolation may not be so accurate with wide range of factors. This can be overcome by developing higher order models, but this increases the number of experimental trials.

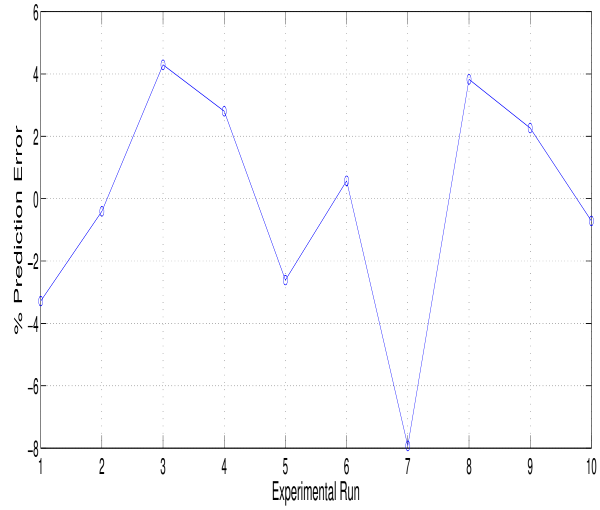

Fig. 1: Validation test results

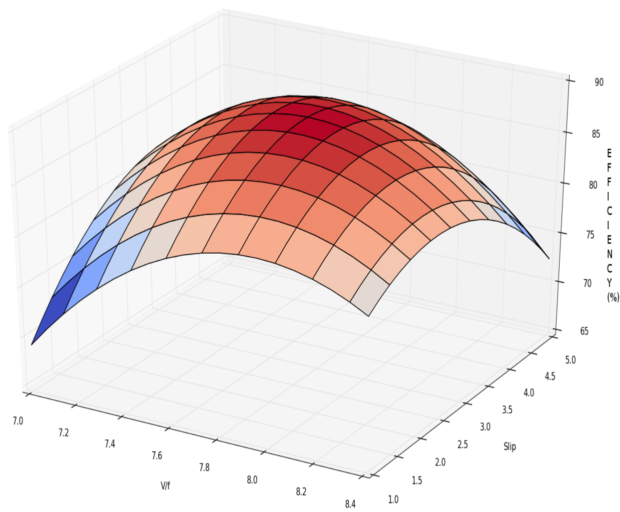

Fig. 2: Response surface plot showing the effect of applied voltage and slip on efficiency

VI. Conclusion

An investigative analysis of parametric influence of voltage to frequency ratio and slip on IM efficiency has been presented in this article. For this purpose, we have made use of the second order RSM based mathematical model of IM efficiency. The data points required to construct the model were determined by carrying out load tests on IM as per FFD experimental layout plan of DOE. The parametric analysis revealed that the efficiency of IM exhibits nonlinear behavior with both voltage to frequency ratio and slip. Also deduced from the experiments is that efficiency is at it's minimum when slip and voltage to frequency ratio are low. Further, the IM efficiency is highly sensitive to variations of slip at lower values of voltage to frequency ratio.

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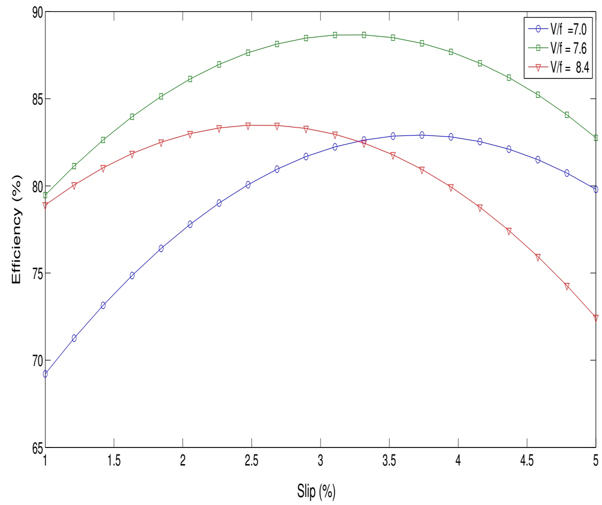

Fig. 3: Direct effects of slip and applied voltage on IM efficiency

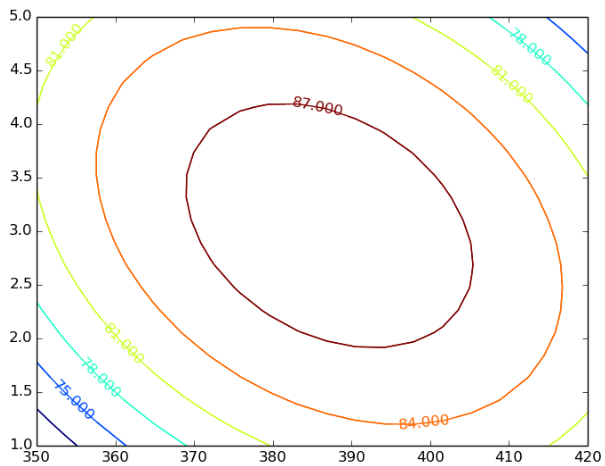

Fig. 4: Contour plot of IM efficiency as a function of applied voltage and slip

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